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Discounting the benefits of climate change mitigation

How much do uncertain rates increase valuations?

Richard Newell
William Pizer
Resources for the Future

Pew Center on Global Climate Change

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Prepared for the Pew Center on Global Climate Change

by
Richard Newell
William Pizer
Resources for the Future
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Foreword *Eileen Claussen, President, Pew Center on Global Climate Change*

How do we compare the costs of greenhouse gas mitigation measures taken today with the benefits produced by these actions in the future? How do we calculate the value of an investment when benefits will continue to accrue over centuries? These are important questions, because the way we value the benefits of mitigation measures will guide us in developing cost-effective solutions to the threat of climate change. This report highlights one important variable in this determination that is often left unexamined in current climate change models—uncertainty in future interest rates.

Underlying existing climate change models is a specific set of assumptions regarding emissions levels, economic growth and flexibility, technological innovation, climate change policies, and the magnitude of climate change damages. Though this set of assumptions varies from model to model, each includes a discount rate, which is used to compare costs and benefits over time. The discount rate tells us how high future benefits need to be to justify spending a dollar today. While there is considerable debate regarding the appropriate discount rate to apply to any cost-benefit analysis conducted across generations, most climate models choose one rate (2-7 percent is a common range) and hold it constant over the time horizon of the model.

This study questions that conventional approach. Rather than assuming that interest rates remain fixed over hundreds of years, authors William Pizer and Richard Newell argue that future rates are uncertain. Using an integrated assessment model of climate change, they demonstrate that acknowledging uncertainty about future rates will lead to a higher valuation of future benefits—regardless of the initial rate one chooses. This is a significant finding, because it reveals that including the effect of interest rate uncertainty could raise valuations by as much as 95 percent relative to conventional discounting at a constant rate. In other words, changing the approach to discounting in this manner results in significantly higher projected benefits of addressing climate change.

This report is the first to be published as a technical report in the Pew Center's economics series. The results of this and other ongoing Pew Center analyses will be incorporated into a dynamic general equilibrium model in order to produce a set of model estimates that better capture the full complexity of the climate change issue.

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Executive Summary

Most environmental policies involve a trade-off between short-term costs and longer-term benefits. Investments in cleaner technologies and abatement equipment, for example, require up front expenditures that produce environmental improvements over time. In cases where a pollutant decays within a few years, the time horizon for analyzing costs and benefits depends on the lifespan of the investment—perhaps as long as 50 or 100 years. By contrast, the benefits of climate change mitigation measures are linked not only to the lifespan of physical capital but also to the lifespan of greenhouse gases (GHGs), which may remain in the atmosphere for centuries. For this reason, climate change presents specific challenges for determining the proper balance between future benefits and present-day costs.

This paper explores some of the analytic difficulties of applying conventional discounting techniques to long-term problems such as global climate change. In particular, the paper focuses on the influence of uncertainty in the discount rate on the valuation of future climate damages, finding that this uncertainty has a large effect on valuations at horizons of 100 years or more in the future. Relative to the standard approach that ignores this uncertainty, the paper finds that the valuation today of benefits 300 years or more in the future rises by a factor of many thousand solely due to this uncertainty in discount rates. The paper also finds that the debate over which is the "right" rate to use is rendered less important once uncertainty in that rate is taken into account.

How do we compare costs and benefits that are separated by many decades or even centuries, thereby involving intergenerational comparisons? Individual experience typically involves trade-offs of at most 20-30 years, as one invests in a new house or saves for retirement. Businesses face decisions with similar horizons as they choose to invest in research and new equipment. In each of these cases, the market interest rate plays a central role: it allows us to convert costs and benefits at different points in time into comparable costs and benefits at a single point in time. This procedure is known as *discounting*.

Discounting is used as a tool for modeling optimal solutions for many long-term problems, including climate change. For example, an integrated assessment model of climate change can be used to estimate the time-profile of benefits associated with the reduction of one ton of carbon emissions in the year 2000. In order to use this information to conduct a policy analysis, the standard approach would be to choose a single interest rate, convert this path of values into equivalent discounted values in 2000 based on the chosen interest rate, and add them up.

Several practical issues complicate the application of this straightforward concept. Current tax policy creates a distinction between the rate of return to corporate investments (10 percent) and the rate of return that is available to individual investors after corporate taxes (7 percent). Focusing on the rate available to individuals, there is also a difference between the rate of return to equity (7 percent), the return to bonds (4 percent), and the return to each of these remaining after personal income taxes. While no consensus exists on the appropriate rate to use for discounting, there is a tendency to focus on the return to bonds (4 percent) for evaluating longer-term policies.

There is concern, however, among economists and non-economists about using conventional discounting techniques to value public benefits over hundreds of years, where trade-offs are evaluated across multiple generations. On the basis of equity, some argue that lower discount rates should be used to compare the value of costs and benefits between generations. Such an argument is common regarding the use of discount rates in the context of climate change modeling, which involves complex projections *centuries* into the future.

Here we focus on *uncertainty* in market interest rates, and evaluate the potential consequences for long-term discounting. Few markets exist for assets with maturities exceeding 30 years, making the interest rate beyond that horizon even more uncertain than the ambiguity among pre- and post-tax returns to bonds and equity. By focusing on the impact of this uncertainty, this study shows, perhaps surprisingly, that uncertainty about future changes in interest rates can have important consequences for the valuation of benefits over distant horizons.

To best understand how future interest rates are likely to change, we examine the behavior of long-term government bond rates in the United States. These rates reveal persistent changes, including a secular decline from near 6 percent in 1800 to 3 percent at the end of the 20th century along with five noticeable shifts of at least 1 percent lasting ten years or more. This leads us to believe that it should not be surprising if persistent changes in interest rates occur in the future, i.e., future rates are uncertain.

We use these data on long-term government bond rates to estimate the uncertainty surrounding interest rates in the past, to simulate uncertain rates in the future, and to compute the appropriate discount factors for various time horizons and alternative base rates. Starting with the assumption that the initial rate should equal the average rate of return to government bonds (4 percent), but that future rates follow an uncertain random trajectory (estimated based on historical data), we find that the discount factor after 400 years is over 40,000 times higher than if we instead assume that the rate remains fixed at 4 percent forever.

When we construct similar measures of the uncertainty effect for alternative initial rates of 2 percent and 7 percent, we find that the effect of uncertainty is larger for higher interest rates—this makes sense because higher rates have a larger opportunity to decline with uncertainty and thereby raise valuations. Comparing the discount factors directly, we find that the valuation of benefits occurring in the future is less sensitive to the choice of the initial discount rate when the effect of uncertainty is taken into account. That is, not only do valuations rise when one considers uncertainty, but they become less sensitive to whether the analysis is based on the after-tax return on bonds or the pre-tax return on equities.

Applied to one estimate of the consequences of climate change, the effect of uncertainty is large. Using the government bond rate of 4 percent, the expected present value of consequences from current carbon dioxide (CO_2) emissions increases by over 80 percent when we incorporate the effect of future interest rate uncertainty. An initial rate of 7 percent yields a 95 percent increase in the value of carbon mitigation, while an initial rate of 2 percent yields an increase of about 55 percent.

The results of this study show that the conventional application of constant discount rates undervalues the benefits of GHG abatement measures. Moreover, they suggest that the concern expressed by those who argue for low discount rates is at least partially addressed—without abandoning conventional economic theory—by viewing future interest rates as uncertain. While this will not yield the same dramatic effects as the decision to arbitrarily apply a lower discount rate, uncertainty does have a large effect on valuations at horizons of 100 years or more in the future.

I. Introduction

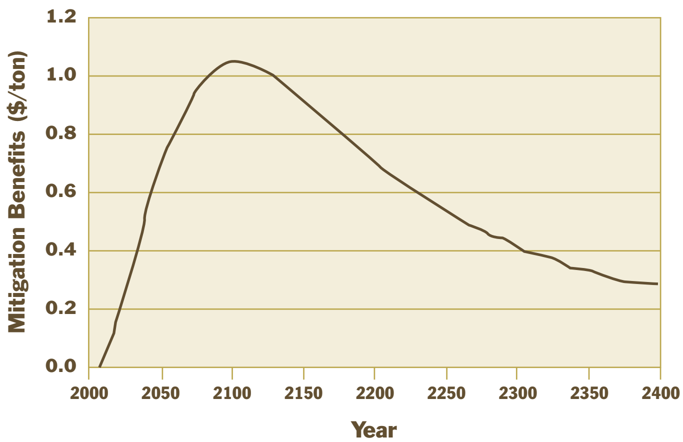
Most environmental policies involve a trade-off between short-term costs and longer-term benefits. Investments in cleaner technologies and abatement equipment, for example, require up front expenditures that lead to environmental improvements over time. For cases where the pollutant decays within a few years, the time horizon for balancing costs and benefits depends on the lifespan of the investment—perhaps as long as 50 or 100 years.[1] Climate change presents a more dramatic need for balancing costs and benefits over time because the benefits of climate change mitigation measures are linked to the lifespan of greenhouse gases (GHGs), not physical capital, and GHGs may remain in the atmosphere for centuries.[2]

How do we compare costs and benefits that are separated by many decades or even centuries? Individual experience typically involves trade-offs of at most 20-30 years, as one invests in a new house or saves for retirement. Businesses face decisions with similar horizons as they choose to invest in research and new equipment. In each of these cases, the market interest rate plays a central role: it allows us to convert costs and benefits at different points in time into comparable costs and benefits at a single point in time. This procedure is known as *discounting.*

When we consider horizons longer than a few decades, we run into trouble. Few markets exist for assets with maturities exceeding 30 years, making the interest rate beyond that horizon even more uncertain than it otherwise might be. Perhaps surprisingly, this uncertainty can have important consequences for the valuation of benefits over distant horizons. As we demonstrate below, our valuation today of benefits 300 years or more in the future rises by a factor of many thousand solely due to this uncertainty in interest rates. Understanding the effect of interest rate uncertainty and quantifying its impact on valuation estimates is the primary purpose of this paper. Note that this is distinct from an analysis of uncertainty in the costs and benefits of climate change mitigation *at the time they occur,* which is a very important, but separable issue.

Figure 1

Time Profile of Benefits from Reducing 1 Ton of Carbon Emissions in 2000



Note: Based on the Dynamic Integrated Climate Economy (DICE) Model (Nordhaus and Boyer, 2000; Nordhaus, 1994).

Despite the potential magnitude of the effect of interest rate uncertainty, virtually all climate models—and indeed most economic analyses of long-term problems—ignore it. These models begin by producing estimates of costs and benefits at the time those consequences occur. The standard approach is to then choose a single interest rate, fixed over the time horizon of the model, and to use that rate to convert values at different points in time into equivalent values in a base year (see Box 1 on how to compute discount factors). Although uncertainty in future interest rates can have huge implications for the results of these models, studies rarely even include sensitivity analysis for this crucial variable. Only recently have economists taken note of the importance of uncertainty in the interest rate itself, or its capacity to change economic analyses in a meaningful way.[3]

For example, an integrated assessment model of climate change can be used to estimate the time-profile of *mitigation benefits*—the benefits associated with reducing one ton of carbon emissions—in the year 2000, as shown in Figure 1. This figure is based on the Dynamic Integrated Climate Economy (DICE) model (Nordhaus, 1994; Nordhaus and Boyer, 2000), which we describe more fully in Box 2.[4] Regardless of any disagreement about the *magnitude* of climate consequences, this figure depicts a sensible pattern of mitigation benefits having a delayed, then an increasing, and finally a declining effect, as the climate adjusts first to the increase in carbon dioxide (CO_2) and then to its gradual decay.[5] It turns out that only the overall shape of the path of benefits, not the magnitude, matters for our analysis of the effect of interest rate uncertainty on the valuation of mitigation benefits.

Box 1

Computing Discount Factors

How are discount factors computed from interest rate paths (and vice-versa)? Discount factors summarize the degree to which one is willing to trade off future costs against costs today. When we associate a discount factor of 0.2 with consequences 40 years in the future, we are saying that $100 40 years in the future would be traded for 0.20 x 100 = $20 today. How does this follow from a sequence of interest rates?

When the interest rate is constant over time, the discount factor is easily expressed as

$$\beta_t = \frac{1}{(1+r)^t}$$

where t is the number of years in the future (40 in our example), β_t is the discount factor (0.2), and r is the constant interest rate (4.1 percent). In other words,

$$0.20 = \frac{1}{(1.041)^{40}}$$

When the interest rate changes over time, due to random fluctuations or natural trends, this simple relation no longer holds. One way to compute the discount factor is by constructing the product,

$$\beta_t = \frac{1}{(1+r_1)} \times \frac{1}{(1+r_2)} \times \ldots \times \frac{1}{(1+r_t)} \qquad (1)$$

Alternatively, one can compute the discount factor recursively,

$$\beta_t = \frac{\beta_{t-1}}{1+r_t} \qquad (2)$$

Note that this can be re-arranged to yield an expression for the interest rate in terms of discount factors in adjacent years,

$$r_t = \frac{\beta_{t-1}}{\beta_t} - 1 \qquad (3)$$

These two expressions (2) and (3) show how one moves back and forth between sequences of interest rates and discount factors.

Note:
A trick used in our simulations is to note that if we convert to the continuously compounded rate, given by $r_t^* = \ln(1+r_t)$, we have $\beta_t = \exp(-\Sigma_{1 \le s \le t} r_s^*)$. This substantially reduces the time required for simulations because a large number of division operations [in Equation (1)] are replaced by addition operations.

In order to use the projections in Figure 1 to conduct a policy analysis, the standard approach would be to choose a single interest rate, convert this path of benefits into equivalent discounted values in 2000 based on the chosen interest rate, and add them up. Applying an interest rate of 4 percent—the average rate of return to government bonds over the past 200 years—one obtains a discounted value of about $6 per ton using the path depicted in Figure 1. This value could be balanced against marginal mitigation costs to determine optimal reductions or could be directly interpreted as the optimal tax on CO_2

Box 2

The DICE Model

The Dynamic Integrated Climate Economy (DICE) model is a stylized representation of the global economy and atmosphere that is useful for understanding the dynamics of climate change.[i] A single consumer represents the world's population. A single production sector, representing global economic activity, uses labor and capital to produce a single good that is partly consumed and partly invested by the consumer each period. An interest rate of about 4 percent leads consumers to balance their current and future consumption, thereby determining investment. Based on the underlying growth assumptions for population and technology, the DICE model generates a growth rate of 2.3 percent for world output in the year 2000, declining to 1.1 percent by 2050.[ii]

GHG emissions, primarily CO_2, are a consequence of economic activity and occur in direct proportion to output in the DICE model.[iii] These emissions accumulate and increase the stock of CO_2 in the atmosphere, typically measured relative to the pre-industrialization level of about 280 ppm (or 590 GtC).[iv] As this stock grows, it increases the amount of solar radiation trapped by the earth's atmosphere. The earth then warms in order to dissipate the additional heat back into space, roughly 3 degrees Celsius for every doubling of the stock.[v]

This warming occurs with a delay. About half the total temperature change due to emissions in the year 2000 occurs by 2015, three-quarters by 2030, and so on. As this warming takes place, the emitted CO_2 is also slowly absorbed into the oceans, reversing the warming effect. About half of the emissions in the year 2000 are absorbed by 2080, three-quarters by 2160, etc. Three hundred years after a ton of CO_2 first accumulates in the atmosphere, roughly 10 percent will remain. *Figure 2* summarizes the resulting path of temperature changes arising from a ton of emissions in the year 2000.

The DICE model posits that damages from global warming are proportional to the square of the temperature change multiplied by the level of global output. That is,

$$D = c \times T^2 \times Y \qquad (4)$$

where T is the temperature change, Y is global output, and c is a constant equal to $1.33\% \div (3^\circ \text{C})^2$. That is, three degrees of warming causes a 1.33 percent loss of global output. Based on the level of global output and the temperature change, one can compute the increase in damages from a further one degree rise in temperature by taking the derivative of this relation: $dD/dT = 2c \times T \times Y$. The result of this calculation for each year over the DICE model's forecast horizon is shown in *Figure 3*.[vi] Importantly, growth in output as well as general increase in global temperatures imply that damages per degree of warming rise over time.

Coupling Figures 2 and 3, one can compute the damages in each year caused by an extra ton of emissions in 2000, as shown in *Figure 1*. These damages represent the product of the estimated temperature change from extra emissions in 2000 (*Figure 2*), multiplied by the damage caused per degree of temperature change (*Figure 3*). In this way, we see that the hump-shaped damage profile arises because there is a delay in the peak temperature change (about 40 years) and the damages per degree of additional warming continue to rise throughout the model horizon (as both T and Y in Equation (4) rise over time). Note that while the DICE model frames the consequences in terms of *damages* per ton of emitted carbon, one can equivalently think in terms of *benefits* per ton of mitigated (e.g., abated) carbon as we do in *Figure 1*. Using the equilibrium interest rate in the DICE model of 4 percent, one can convert the value of future benefits into their equivalent value in the year 2000 and then add them up. This works out to be about $5.70.[vii]

Notes:

i There are many nuances to the DICE model ignored in this explanation—in particular, GHG abatement and the potential to optimize over both savings and emissions level.

ii The interest rate falls to 3 percent after two hundred years as productivity growth declines to zero.

iii Emissions per unit of output begin at about 0.37 GtC (gigatons carbon) per $trillion in 2000, falling to 0.27 GtC per $trillion in 2050.

iv Current level of CO_2 in the atmosphere is 368 ppm (Keeling and Whorf, 2000). The conversion factor from ppm (parts per million by volume) to GtC is 2.13 GtC/ppm, based on an atmospheric mass of 5.14×10^{18} kg and a mean molecular weight of 29.

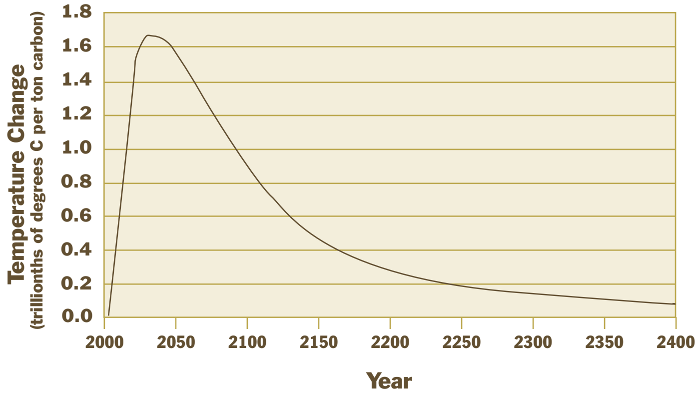
v The most recent report of Working Group I of the Intergovernmental Panel on Climate Change revised their forecast range of warming over the next century from 1.0-3.5 ºC to 1.4-5.8 ºC (IPCC, 2001).

vi Output in 2000 is about $28 trillion and the temperature change in 2000 relative to pre-industrialization is 0.8 ºC based on the DICE model.

vii Nordhaus (1994) reports optimal carbon taxes of $5.29 in 1995 and $6.77 in 2005 (Table 5.7).

Figure 2

Influence of **Carbon Emissions** on Temperature Change



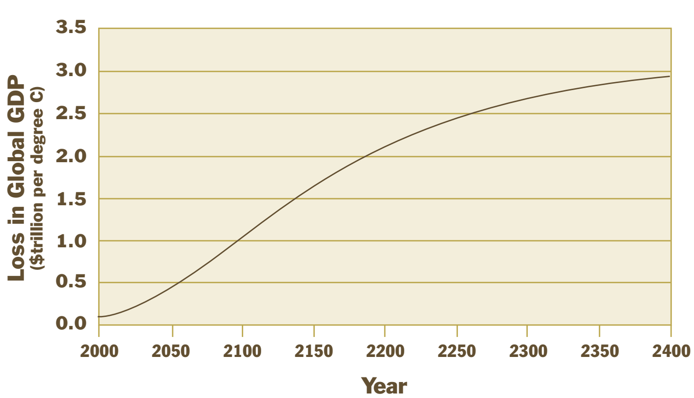
emissions (if it is assumed that mitigation benefits remain proportional to the reduction in emissions).

Despite the appealing simplicity of this approach, no one really knows for sure that the interest rate will be 4 percent for the next 400 years. One could instead adopt the attitude that the interest rate is known in the short term, but is increasingly uncertain the farther one looks into the future. Using past variation in interest rates to inform our opinion about future uncertainty, we demonstrate in this paper that the expected discounted value of this particular path of mitigation benefits is really about $10. This 80 percent increase in the estimated benefits of CO_2 emissions abatement is entirely a consequence of treating future interest rates—and nothing else—as uncertain. Furthermore, the percentage increase depends only on the time profile of benefits, not the magnitude.

In Section II of the paper we discuss the general economic rationale behind discounting; in Section III we describe both the conceptual and empirical influence of interest rate uncertainty on future valuation; in Section IV we apply our computation of uncertainty-adjusted discount factors to the problem of valuing future climate change consequences; and in Section V we summarize and conclude.

Figure 3

Influence of **Temperature Change** on Economic Loss



II. A Plethora of Discount Rates

Before turning to the specific question of how interest rate uncertainty influences our valuation of policy consequences in the distant future, we first examine why one discounts the future at all, as well as which rate or rates ought to be the basis for that discounting. As we will see in Section III, regardless of which rate(s) are used—the influence of uncertainty remains quantitatively and qualitatively important.

The term *discounting* arises in the context of valuing future consequences because individuals typically value future consumption less than present consumption—they *discount* the future. Over horizons of less than 30 years, discounting is inextricably tied to the interest rate, which reflects the rate at which income and wealth can be traded across time. For example, investing $100 at a 7 percent interest rate yields $107 at the end of the year. The notion that $100 today can be traded for $107 next year indicates an equivalence between these values—at least from the market's perspective: $107 next year is valued at only $100 today. One would say that the future (next year) is *discounted* at 7 percent.

The traditional investment criterion used in business decisions provides another way of looking at the rationale behind discounting. Let's say private firms can borrow funds at a 7 percent interest rate. From their perspective, any investment project that returns more than 7 percent per year is profitable. That is, if $100 invested in research or equipment yields a net pay-off of more than $7 every year, they can pay back creditors and have something left over. Otherwise, they lose money. This is a statement of the *net present value rule:* if the stream of net benefits from a project discounted at 7 percent is greater than zero, then the project is desirable. For this reason, the borrowing rate (7 percent in this example) is often referred to as the "hurdle rate" for private investment projects.

A. Market Rates

In reality, 7 percent is approximately the real return to investment in large companies over the period 1926-90[6] and is the rate recommended by the Office of Management and Budget (OMB) for standard cost-benefit analysis.

Box 3

U.S. Government Policy on Discount Rates

According to the Office of Management and Budget (OMB) Circular No. A-94, "Constant-dollar benefit-cost analyses of proposed investments and regulations should report net present value and other outcomes determined using a real discount rate of 7 percent. This rate approximates the marginal pre-tax rate of return on an average investment in the private sector in recent years." In contrast to public investments and regulatory analysis, the Circular states that cost-effectiveness, lease-purchase, internal government investment, and asset sales analyses should be evaluated using the Treasury's borrowing rates (about 4 percent). The exact rationale appears to be the difference between valuing costs and benefits *external* to the government in the first case, which are compared to private investments, versus costs and benefits *internal* to the government in the latter case, which are evaluated at the rate of government borrowing.

Both the Circular and a later OMB document, Economic Analysis of Federal Regulations Under Executive Order 12866, also referred to as the "best practices" document, point to an alternative for benefit-cost analyses of proposed investments and regulations based on a lower discount rate. Specifically, the consequences of the investment or regulation should be converted into effects on consumption (versus investment) and then these consumption effects should be discounted using a consumption rate of interest—the rate faced by consumers when they save, rather than businesses when they borrow. The main difference between these rates is the tax on investment income, as discussed in the text. Agencies must seek OMB approval before using this alternative, referred to as the shadow-price approach.

The Treasury's borrowing rate of about 4 percent or lower is often used as a consumption rate of interest because U.S. government bonds reflect the lowest-risk investments available (Lind, 1990; Lyon, 1990; Hartman, 1990). However, both the OMB Circular and best practices documents highlight that it may be difficult to compute the consumption consequences of a proposed policy, and that the Treasury rate may not be the best measure of the rate faced by consumers because few individuals actually invest in Treasury bonds.

More recently, the EPA has developed its own guidelines (U.S. EPA, 2000). These guidelines state that a consumption rate of interest of between 2 and 3 percent should be applied directly to costs and benefits, without converting to consumption effects. This is based on the findings of recent studies of capital markets that indicate a high degree of capital mobility (Lind, 1990). Under such an assumption, the conversion of costs and benefits into consumption effects is one-for-one, and the shadow-price approach would simply apply the consumption rate of interest to all costs and benefits. Their recommendation of a 2 to 3 percent rate (versus 4 percent in the OMB Circular) seems to be based on a slightly different view on the historic return to bonds and/or adjustment for inflation. The Congressional Budget Office and the General Accounting Office also favor the use of discount rates based on government bond rates (Lyon, 1990; Hartman, 1990).

For practitioners outside the purview of OMB, however, there is regrettably little consensus on the appropriate discount rate—even among other government agencies (see Box 3, U.S. Government Policy on Discount Rates). Part of the problem arises because taxes create a wedge between the observed market return to private investments and the eventual return to consumers after taxes.[7]

Corporate income taxes of 35 percent, for example, imply that the pre-corporate-tax return to private investment is closer to 11 percent. Personal income taxes, on the other hand, imply that the return for ordinary people is closer to 4 percent because individuals pay up to 50 percent in federal, state, and local income taxes on their 7 percent pre-personal-tax return. This fully after-tax rate of return—or *consumption rate of interest*—is taken as a measure of the rate at which people trade off their spending over time. Individuals face the option of consuming today versus foregoing such consumption, investing the money, and consuming the proceeds from the investment at some future date. The consumption rate of interest is the rate at which they can perform this shift in consumption over time.

The consumption rate of interest, however, is difficult to pin down. For a particular individual, it will depend on the individual's income tax bracket, the extent to which they utilize tax-deferred savings options, whether or not they are a net saver or borrower, and the types of investment and debt they employ. The availability and use of investments with varying degrees of risk raises another complication, because riskier assets (such as equities) compensate investors for this risk by paying a higher rate of return than less risky assets (such as bonds).[8] Thus, while equities have had an average return of about 7 percent, bonds have had an average return of only 4 percent before- and 2 percent after- taxes. Because the risks involved in social projects may not be comparable to commonly observed market risks, one may want to value that risk differently. In this context, it is useful to separate the issue of risk from the issue of discounting.[9]

To separate these issues, we need to identify low-risk investments. Most people consider government bonds to be the safest possible investment and therefore economists point to the return on government bonds as a benchmark for the consumer interest rate absent any risk premium. For this reason the government bond rate is the discount rate favored by the Environmental Protection Agency, the Congressional Budget Office, and the General Accounting Office (as described in Box 3). On the other hand, some economists consider bonds to be unrepresentative assets and prefer to focus on the average return to equity as the appropriate rate.[10]

Confronted with the choice between the consumption rate of interest and the rate of return to private investment, we focus on the former. Because our concern with climate policy decisions is ultimately regarding the future welfare of people—not firms—the consumption interest rate is more appropriate. To the extent that there are effects on firms rather than on individuals, these effects need to be converted into their consumption consequences for individuals. These consequences can then be valued along with direct consumption effects at the consumer interest rate. Converting firm effects into consumption effects is referred to as the "shadow price of capital" approach because any consequences on firm capitalization are valued in terms of their consequences on consumption—or "shadow value" to consumers.[11]

B. Intergenerational Rates

At this point, there are plausible arguments for the use of consumption discount rates of between 2 percent and 7 percent, depending on the personal income taxes faced by consumers and one's view about risk and the appropriate asset to reference.[12] Based on a particular set of assumptions concerning taxes and the reference asset, one can then turn to the market in order to observe the *current* rate.[13] *Future* rates based on bonds (at the lower end of this range) can be inferred from bonds with longer horizons, up to about 30 years. Beyond this time horizon, however, two problems arise. First, few low-risk securities exist with maturities of more than 30 years, leaving longer-term interest rates uncertain.[14] Second, at horizons beyond 30 years, our decisions influence not only the current generation but future generations as well.

Considering this problem, many economists dating back to Frank Ramsey (1928) have argued that it is ethically indefensible to discount the *utility* (i.e., well-being) of future generations—although this does not imply a zero discount rate for their *consumption* (e.g., measured in dollars).[15] Others have argued that at a minimum, the intergenerational discount rate need not equal the observed rate used by individuals within their own generation.[16] These arguments have lead to an unfortunate tendency to simplify the problem by applying lower rates over longer horizons. Such an approach may create additional problems, however, as long horizons eventually become short horizons (see Box 4).[17]

Low or zero rates of intergenerational discounting have far-reaching consequences for public policy beyond the realm of climate change mitigation. One implication is that an excellent government policy would be to tax the current generation and use the revenue to invest in private capital markets for the benefit of future generations. With pre-tax returns of greater than 7 percent, such investments would make future generations immensely better off. Another implication would be that all government-funded activities with long-term impacts such as medical research, infrastructure development, and environmental measures ought to be evaluated with these same low or zero rates. Yet, it is questionable whether the public desires—or if enough funds exist—to pursue all the public activities that evaluate favorably at these rates.

The thrust of this paper is to understand how uncertainty about future interest rates influences our valuation of future benefits, and how selecting a rate based on ethical concerns would tend to

Box 4

Time-Inconsistency and Hyperbolic Discounting

The conventional economic approach to policy and project evaluation assumes people discount the future at a constant exponential rate. That is, the value in one period equals a constant (less than one) times the value in the next future period—and that constant is always the same. As a result, the amount of discounting that takes place between two points in time depends only on the absolute time interval separating them, not *when* they occur. Empirical research has shown, however, that individuals often express preferences implying the use of a discount rate that declines as they look further into the future (Ainslie, 1991; Cropper et al., 1994). Such a declining rate of discount is frequently referred to as "hyperbolic" discounting, in contrast to conventional, exponential discounting.

However, the use of a rate that declines with certainty, though perhaps reflective of individual preferences, would produce "time-inconsistent" decisions. For example, suppose you decide to use a 5 percent rate for 100 years into the future and 0 percent afterward. From that perspective in the year 2000, a choice that trades a $1 loss in 2150 for a $2 gain in 2200 is desirable, because there is no discounting between these periods. After 2050, this choice begins to look worse and worse, as the interval between 2150 and 2200 begins to be discounted. Eventually, in the year 2065, you will want to reverse your initial decision and choose the $1 in 2150. Recognizing that this will assuredly happen represents a time inconsistency. That is, the mere passage of time will make you want to change your choice. Thus, even though you might "like" to act as if you discounted hyperbolically, in reality this type of behavior cannot persist.

Because it only makes sense to evaluate policy decisions along behavioral paths that will actually be carried out, a natural question is what type of behavior would we expect from individuals with hyperbolic preferences? Interestingly, Laibson (1996) showed that someone with hyperbolic preferences should make decisions over time that look exactly the same as someone who uses conventional exponential discounting (see also Arrow, 1999). And the rate at which they discount should equal the rate of return to capital (see earlier discussion). Some have suggested that one should therefore exponentially discount the future using the market interest rate whether people have hyperbolic or exponential preferences over time (Cropper and Laibson, 1999).

But where does this leave our result that effective discount rates should decline in the future, once proper account is taken of uncertainty in the discount rate itself? First note that our model with discount rate uncertainty is based fundamentally on the concept of exponential discounting at the market rate of interest. In contrast to the earlier idea of declining deterministic discount rates, declines in our effective discount rate are not time inconsistent. Any desire to revise a choice made in the past is no longer determined by the mere passage of time; rather, it is determined by the revelation of uncertainty about interest rates. In an uncertain world there is always the possibility that *ex ante* good decisions turn out to be regrettable *ex post,* once nature has revealed herself. But the *ex ante* decision was still the best course of action when the decision had to be made, and thus it reveals no inconsistency in the decision framework, unlike the time-inconsistency problem posed by hyperbolic discounting. In other words, behavior that would be time-inconsistent in a deterministic world is legitimate state-contingent behavior in a world with uncertain discount rates.

eliminate that uncertainty.[18] If one wants to discount based on consumption growth (see endnote 15), uncertainty remains and the effect could be gauged by our study of a 2 percent discount rate on consumption—the lower end of the range of rates we actually explore.[19] More generally, however, the ethical decision to use a low rate will overwhelm any effect owing to uncertainty.

Because these concerns over intergenerational equity can be used to defend an arbitrarily low rate and eliminate the effect of uncertainty, we remain focused on the use of market rates between 2 percent and 7 percent. Even if one is attracted to the idea of discounting based on intergenerational equity, it can still be interesting to understand the consequences of uncertainty in a more conventional framework. As we will see, our analysis implies that correctly handling uncertainty lowers the effective discount rate in the future in a way that all generations after a certain horizon are essentially treated the same. In that sense, our paper can be viewed as an argument for intergenerational equity that originates from conventional neoclassical economics.

C. Choosing a Rate

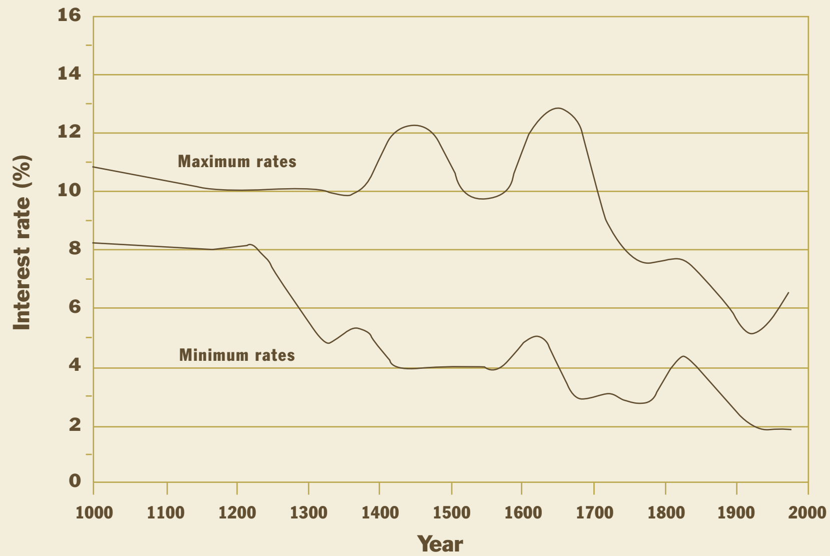
Intergenerational issues aside, the problem of valuing the future consequences of a far-reaching policy such as climate change mitigation remains vexing. The plausible range of discount rates extends from 2 percent to 7 percent, which has a corresponding difference in discounted values of 200-million to one looking 400 years into the future. Beyond the rules of thumb adopted by various government agencies—which themselves span this range—there is little justification for narrowing our consideration despite an enormous volume of writing on the topic.[20] On top of this enormous degree of variation depending on the choice of rate, we are here to consider the further effect of uncertainty about how these rates may change in the future.

Ironically, we find that this uncertainty about changes in future interest rates *reduces* the valuation dilemma. Intuitively, uncertainty about future rates implies that the initial choice of rate is less important as random variation will mix up those choices over time. In the next section we show that the difference in valuation between using a 2 percent versus a 7 percent discount rate is only a factor of about 40 to one after 400 years, once one considers uncertainty about those rates in the future. Although this is still a significant difference, it is obviously much smaller than 200-million to one.

For practical purposes, we will focus on an initial rate of around 4 percent, because this lies in the middle of the 2-7 percent range and corresponds to the average rate of return for government bonds, the interest rate for which we have the best data and which is the basis of our empirical work. However, we readily concede that this is somewhat arbitrary and will discuss the range of results produced by using initial values of both 2 percent and 7 percent. Because the academic literature has largely failed to narrow the range further, we believe our own puzzling over the choice of rates will be of limited value—

Box 5

A Millennium of Market Interest Rates



Using data from Homer and Sylla (1998), this figure shows the 50-year minimum and maximum recorded yields on long-term debt for the United States (1800s and 1900s), England (1700s), and several European countries for the 11th to 17th centuries. A persistent increase can be seen at the beginning of the 1600s (and perhaps around 1400, 1800, and 2000) while, generally, there has been a secular decline from rates hovering at 10 percent in the year 1000 to rates closer to 5 percent by the year 2000. This is consistent with the patterns observed over the last 200 years in the U.S.—persistent changes overlapping an apparent long-term trend. Further, the volatility over the past millennium, as recorded by the spread of the minimum and maximum values, has varied considerably—also consistent with the recent U.S. experience.

While there is a temptation to dismiss the "distant" past as irrelevant to our forecast of future interest rate behavior, it is important to recognize the similar "distance" we face in precisely these forecasts. Although plague, famine, and empire-building may be (hopefully) relics of the past, who is to say what global catastrophes might lie ahead? At the same time, it is instructive to recognize that such an instinct to dismiss the distant past is, in effect, a subjective statement in favor of a model based on permanent changes in interest rate behavior, versus a model where deviations eventually vanish and interest rates revert to an unchanging long-run mean (what we characterize as a preference for the random-walk versus mean-reverting model).

most practitioners either have a rate they prefer or will likely consider a range. Instead, we now turn our attention to the issues of how interest rates may change in the future, why these fluctuations raise future valuations, how historical data can quantify this effect, and, finally, how this effect changes the estimated benefits of climate change mitigation.

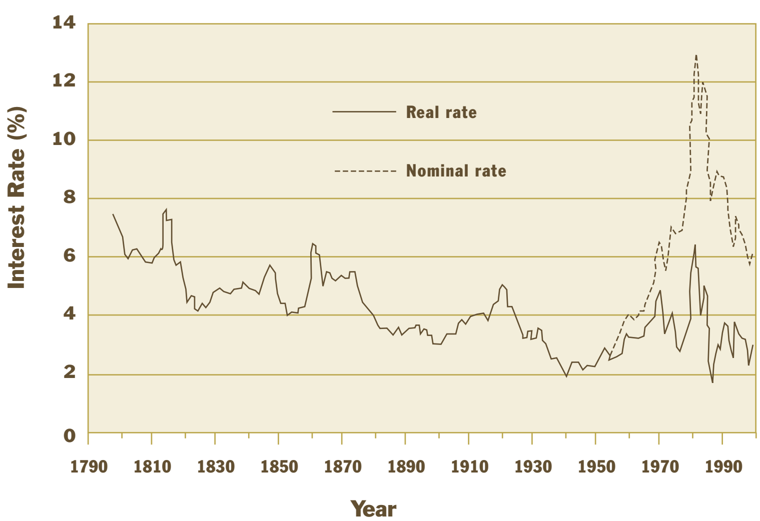
D. Changes in Interest Rates over Time

We have already noted that there are few if any observable market rates for investment horizons more than 30 years in the future, making the interest rate beyond those horizons even more uncertain than it otherwise might be. This begs the question: What sort of assumptions should one make about rates in the future? Our belief is that the past behavior of interest rates over long periods of time ought to form the basis for predicting their future behavior.

To this end, consider the history of interest rates available on long-term U.S. government bonds, plotted in Figure 4. We have chosen to focus on long-term government bonds because they represent the highest quality, lowest risk, market investment consistently available in the United States over the past 200 years. Effective markets for high quality corporate debt and equity, as well as short-term government bonds, did not really exist prior to 1900. For the most part, our series consists of rates on Treasury bonds, except for periods where there were no bonds outstanding or when the market was distorted by changes in banking policy. Those periods are represented by rates on high-quality municipal bonds.

Figure 4

Market Interest Rate on U.S. Long-Term Government Bonds (1798-1999)



Several features stand out in this graph. Most importantly, there have been persistent fluctuations in the real interest rate over the past 200 years. Periods of relatively high rates during the 1810s, 1830-40s, 1860-70s, 1910s and 1970-80s have all been followed by significant declines of 1 percent or more that remain for a decade or more. A secular decline also appears, with rates above 6 percent at the beginning of the series reduced to around 3 percent in recent years. We see inflation appearing in the 1950s, with dramatic differences between real and nominal rates beginning in the 1970s.[21] Nonetheless, fluctuations in the real interest rate over the past 50 years are not dissimilar from the fluctuations in the first half of the 18th century (or over the last millennium—see Box 5).[22]

What does this tell us about future interest rates? Generally speaking, it should not be surprising if interest rates continue to change. Given that we observe a 3 percent decline in rates in the past, it should not surprise us if we see a change of 3 percent in the future—up or down. The real objective at this point is to more precisely quantify future interest rate behavior and evaluate the impact of this behavior on future valuations.

III. The Effect of Discount Rate Uncertainty on Future Valuations

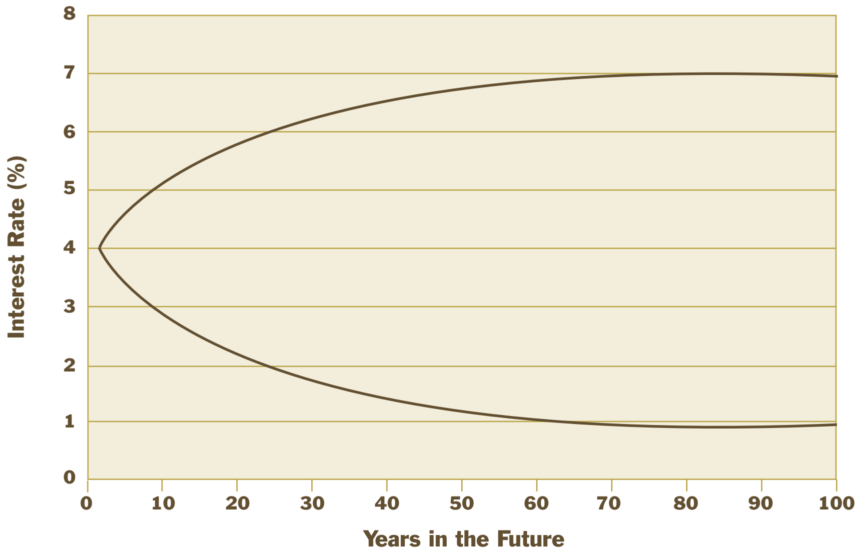
Relative to the standard approach using constant discount rates, a model incorporating uncertainty about future rates will lead to a higher valuation of future benefits, regardless of the initial rate chosen—this is the finding of this paper. In Section III.A. we explore the magnitude of this effect based on historical data, but, first, let us consider the rationale behind these findings. Suppose, for example, that we all agree that the current discount rate should be about 4 percent, perhaps based on the average market return to government bonds. Arguably, the rate in the future might decline to as little as 1 percent over the next 100 years—or rise to 7 percent. Figure 5 shows these two alternate future paths, one declining to 1 percent and one rising to 7 percent.

These alternative paths have important implications for how much one would value consequences of today's actions 100 years in the future. In particular, using the rates shown in the lower path, $100 in 2100 is worth $20.28 today. However, using the rates shown along the higher path, the same $100 in 2100 is worth only $0.20 today. Placing equal weight on these two outcomes, the expected value of $100 in 2100 would be $10.24.

Now here's the interesting point.[23] Suppose we did the same analysis for the present value of $100 in 2101. Based on the lower rate of 1 percent in 2100, the $20.28 would be worth only $20.08 when moved one year further into the future (20.28 ÷ 1.01 = 20.08). Similarly, based on the higher rate of 7 percent in 2100, the $0.20 would decline to $0.19 (0.20 ÷ 1.07 = 0.19). Averaging these results, the expected value of $100 delivered in the year 2101 would be $10.13. Note that the expected value declined by 1 percent (10.24 ÷ 10.13 = 1.01). That is, with equal probability placed on the low path and the high path, the effective discount rate is very close to the lower value, not the average of the two values as one might think at first. As it turns out, it is discount factors, not discount rates, that one should average—and this distinction makes a big difference, especially for long time horizons.

Figure 5

Future Interest Rate Possibilities



Why does one effectively use the low rate rather than the average? Intuitively, discounting benefits 100 years hence only depends on the lower rate because the higher rate discounts future benefits to such an extent that they add very little to the expected value. The expected value of $10.24 almost equals the value when the rate falls to 1 percent, $20.28, multiplied by the probability of that outcome, 50 percent. In fact, the expected value would be virtually the same if the high value interest rate were 10 percent—or 110 percent—instead of 7 percent. In this way, the change in value between periods comes to depend solely on the lower rate.[24]

As noted in the previous section, this has important consequences when it comes to choosing the "appropriate" discount rate. Most analyses will emphasize the enormous difference in discounted values based on discounting at the 2 percent after-tax return to bonds rather than the over 7 percent pre-tax return to equities. If uncertainty about both rates has the same lower bound, namely a zero rate of interest, the uncertainty-adjusted difference in discounted values is limited. As we show in Section IV, this difference is still significant, but *much, much* smaller than with simple constant-rate discounting.

A. Modeling the Long-term Behavior of Interest Rates

Now that we understand the potential impact of uncertainty about future rates, the obvious question is how much it really matters. To that end, we would like to use information about past interest rate fluctuations, such as those on U.S. long-term government bonds in Figure 4, to quantify the likely range of interest rate paths in the future, such as those in Figure 5. In order to do this, however, we have to make some assumptions about interest rate behavior.

One of the more interesting issues is whether we believe interest rates can be zero or negative for an extended period of time. Historically, the rates we examine rarely drop below 1 percent, but some might argue that a downward trend exists over the past millennium and, projecting it forward, interest rates will eventually decline to zero. Philosophically, the question of zero or negative rates has been debated for some time.[25] A strong argument against negative rates is the availability of money, gold, and other durable commodities that retain value with a zero rate of return. When such savings alternatives exist, negative investment returns cannot persist because consumers would cease investing in capital, thereby reducing the capital stock and eventually raising the return to investment.

When one considers the interest rate series in Figure 4, however, it is easy enough to imagine the rate wandering in such a way that it becomes permanently negative in another 100 years or so—unless we place some structure on interest rate behavior. Because we think persistently negative rates are implausible, when we estimate and forecast the model we make a simple adjustment: we construct a model of logged (logarithmic) interest rate behavior. Working with forecasts of logged rates, it is impossible for the rates themselves to be negative. As logged rates become lower, the rates themselves simply get infinitesimally closer to zero—but remain positive.

A second decision one has to make about modeling interest rate behavior is whether to model the underlying determinants of the interest rate—such as economic productivity and consumer preferences—or directly model the interest rate itself.[26] The former, structural approach is appealing because it captures the real workings of the economy, not just the isolated behavior of the interest rate. However, the structural approach requires simplifying assumptions that fix certain aspects of technology and consumer preferences—aspects that are likely to change over the course of decades or centuries. The structural approach also requires having some random, underlying behavior variable to explain interest rate fluctuations.

The alternative we embrace is to focus directly on the interest rate itself. We ask how the future interest rate can be forecast best from the previous history of interest rates and how much unpredictable error exists in each period. The advantage of this approach is that it is not limited by the availability of the other data required to estimate a more complex model. It also does not fix any preference or technology relationships. Instead, it simply assumes that the past patterns of variation and persistent changes in the interest rate will continue in the future. Because it is this variation and persistence that creates the uncertainty effect we are addressing, a simple focus on quantifying these features seems a better strategy than a more complex, and less transparent, structural model.

B. Random Walks

The final modeling question we confront is whether to model interest rates as following some type of ***random walk****—where current rates represent the best estimate of future rates—or as eventually reverting to a long-run mean.* Of course, the term random walk is more frequently applied to phenomena such as stock prices—or drunks. Stock prices are often described as a random walk because their movement up or down over time is random and, whatever their level at a particular moment, that is usually the best guess about their level in the future (plus some average return). Similarly, as a drunk meanders down the street, his movement is random. Wherever he is, that is the best guess of where he is likely to be in the future. Two things therefore characterize random walks: random movements and the proposition that the current location or level is always the best predictor of future location or level. This can be contrasted with the concept of *mean-reversion*—where the random movement remains but the future level is expected to return toward some long-run mean. In such cases, the best predictor of future levels is always the long-run mean (once the current random deviation dissipates).

Now consider interest rates. Is there a long-run mean toward which rates will always return? Or, do recent rates form the best guess of likely future rates? Asked another way, if a decline in interest rates was observed over the next 100 years to the point where they fluctuated around 0.3-0.8 percent, rather than 3-8 percent, what would the best prediction be about the subsequent 100 years? What if they rose to 10-20 percent?

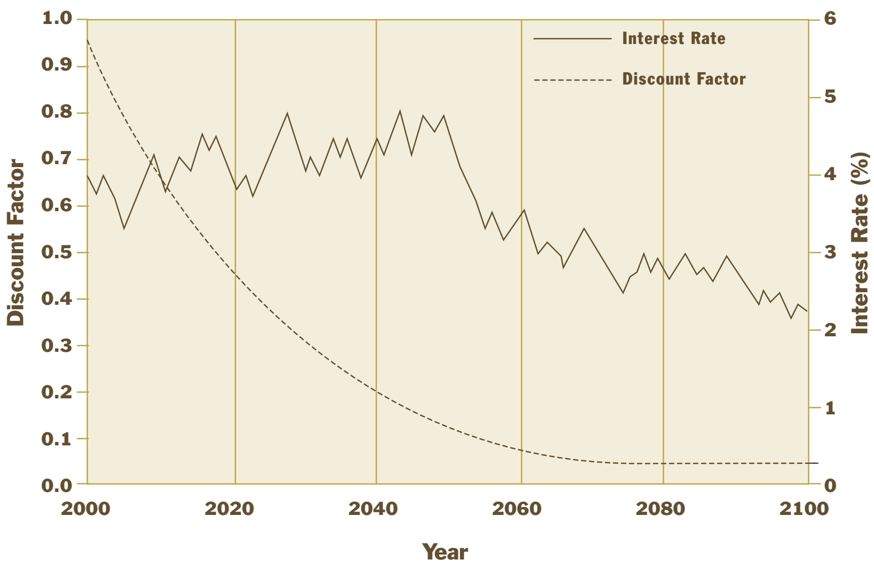
If the answer to this question is that interest rates over the past few decades are the best predictors of future rates, this represents a belief in the persistence of interest rate fluctuations—the belief that there is no fixed long-run mean. Unfortunately, the data neither confirm nor reject this belief. In fact, if one initially gives equal confidence to the random walk and mean-reverting models, and then uses the data to revise those opinions, one would come out with roughly 60 percent odds on the random walk model—only a small change in beliefs in favor of the random walk model.[27]

The choice of models makes an enormous difference, however, when one considers forecasting the future effect of uncertainty. Because the mean-reverting model assumes that interest rates cannot remain high or low indefinitely, the kinds of paths shown in Figure 5 are much less likely. Uncertainty about future interest rates therefore has a much smaller effect on future valuation based on the mean-reverting model. Because we find it unappealing to force the mean rate over the next 400 years to equal the estimated mean rate over the last 200 years, we focus our discussion on the random walk model while presenting results from both models.

Once we have quantified the uncertainty visible in historical interest rates (Figure 4) using either the random walk or mean-reverting model of interest rate behavior, we can use that model to simulate future interest rate paths. While we provide a more detailed description of both estimation and simulation in Appendix A, the intuition behind our simulations is easy. In the simplest random walk version, one can imagine randomly adding either +0.06 or –0.06, with equal probability, to the logged interest rate each period.[28] Starting with a bond return of 4 percent today, one randomly drawn path of interest rates following this behavior is shown in Figure 6.[29] In this figure we show the corresponding (unlogged) interest rate and discount factor. Now imagine repeating this process tens of thousands of times, each time drawing a different, random path. This is how we conduct our simulations of future interest rates.

Figure 6

A Random Walk Interest Rate and the Associated Discount Factor



IV. Valuing Future Climate Change Consequences

At this point, we can use our simulations of future interest rates to analyze the effect of interest rate uncertainty on climate change policies. Our goal in Section III has been to understand how to make future benefits comparable with current costs. Mechanically, the value today of some benefit in the future equals the valuation in the future, times a discount factor that translates future dollars into today's dollars. If one knows the valuation in the future, say $100 delivered 400 years in the future, the problem is determining the right discount factor.

From the aforementioned simulations, we now have tens of thousands of equally plausible values for the discount factor at a particular point in the future. Common sense—and indeed economic theory—would tell us to *average them.*[30] Doing this delivers an expected discount factor and, multiplied by the valuation in the future, an expected present value.[31]

A. Expected Discount Factors

Table 1 presents our estimates of discount factors over the next 400 years based on a 4 percent rate of return in 2000 and using our historical data on long-term government bonds to quantify interest rate uncertainty.[32] We report results for both the random walk model as well as the mean-reverting model. For comparison, we present discount factors associated with a constant rate of 4 percent. Discount factors are expressed in terms of the value today of $100 provided at various points in the future, that is, the discount factor multiplied by 100. Table 2 presents a similar analysis for the alternative rates of 2 percent and 7 percent.

After only 100 years, conventional discounting at a constant 4 percent undervalues the future by a factor of 3 based on the random walk model of interest rate behavior. After 200 years, the future is undervalued by a factor of about 40. That is, while conventional discounting values $100 in the year 2200 at 4 cents, the random walk model values the same $100 at $1.54—about 40 times higher. Going further into the future, conventional discounting is off by a factor of over 40,000 after 400 years.

Table 1

Value Today of $100 in the Future

Years in future	Discount rate model: Constant 4%	Discount rate model: Mean reverting	Discount rate model: Random walk	Value relative to constant discounting: Mean reverting	Value relative to constant discounting: Random walk
0	$100.00	$100.00	$100.00	1	1
20	45.64	46.17	46.24	1	1
40	20.83	21.90	22.88	1	1
60	9.51	10.61	12.54	1	1
80	4.34	5.23	7.63	1	2
100	1.98	2.61	5.09	1	3
120	0.90	1.33	3.64	1	4
140	0.41	0.68	2.77	2	7
160	0.19	0.36	2.20	2	12
180	0.09	0.19	1.81	2	21
200	0.04	0.10	1.54	3	39
220	0.02	0.06	1.33	3	75
240	0.01	0.03	1.18	4	145
260	0.00	0.02	1.06	5	285
280	0.00	0.01	0.97	7	568
300	0.00	0.01	0.89	11	1,147
320	0.00	0.01	0.83	16	2,336
340	0.00	0.00	0.78	26	4,796
360	0.00	0.00	0.73	43	9,915
380	0.00	0.00	0.69	74	20,618
400	0.00	0.00	0.66	131	43,102

Table 2

Sensitivity of Valuation to Initial Interest Rate

Years in future	2% initial rate: Random walk	2% initial rate: Constant rate	2% initial rate: Ratio	7% initial rate: Random walk	7% initial rate: Constant rate	7% initial rate: Ratio
0	100.00	100.00	1	100.00	100.00	1
20	67.54	67.30	1	26.89	25.84	1
40	46.48	45.29	1	8.67	6.68	1
60	33.05	30.48	1	3.52	1.73	2
80	24.42	20.51	1	1.75	0.45	4
100	18.76	13.80	1	1.02	0.12	9
120	14.93	9.29	2	0.67	0.03	22
140	12.25	6.25	2	0.47	0.01	62
160	10.32	4.21	2	0.36	0.00	181
180	8.89	2.83	3	0.29	0.00	557
200	7.81	1.91	4	0.24	0.00	1,778
220	6.97	1.28	5	0.20	0.00	5,851
240	6.30	0.86	7	0.17	0.00	19,726
260	5.77	0.58	10	0.16	0.00	67,829
280	5.33	0.39	14	0.14	0.00	236,788
300	4.97	0.26	19	0.13	0.00	837,153
320	4.66	0.18	26	0.12	0.00	2,992,921
340	4.40	0.12	37	0.11	0.00	10,804,932
360	4.18	0.08	52	0.10	0.00	39,298,213
380	3.99	0.05	74	0.10	0.00	143,866,569
400	3.83	0.04	105	0.09	0.00	529,656,724

The mean-reverting model produces less dramatic yet still significant results, raising the discount factor by a multiple of about 130 after 400 years.

Table 2 presents an alternative comparison using initial interest rates of 2 percent and 7 percent—what one might think of as upper and lower bounds on the consumer rate of interest. We use the same assumptions about random disturbances estimated from data on government bond rates, but initialize the random walk at a different rate.[33] Again, we compute discount factors based on the corresponding constant rate as a benchmark. When we compare the ratio of random-walk to constant-rate discount factors, these valuations show that the *relative* effect of interest rate uncertainty (measured by this ratio) rises as the initial rate rises. For example, the effect at a horizon of 400 years raises the valuation by a factor of 530 million based on a 7 percent rate. Meanwhile, the effect over a similar horizon is a factor of a little over 100 based on an initial 2 percent rate (from the bottom line of Table 2). Intuitively, the effect must be smaller for low discount rates (e.g., 2 percent) because the range of possible lower rates (0-2 percent) is narrower. It is the possibility of these lower rates that raises the valuation.

As we noted earlier, these effects imply that the difference between valuations using different initial rates is generally *smaller* when uncertainty about future rates is incorporated. Note that the ratio of discount factors based on the random walk model, but starting with initial rates of 2 percent versus 7 percent, is a factor of about 40 after 400 years (see bottom line of Table 2: 3.83 ÷ 0.09 ≈ 40). Though obviously a factor of 40 is still substantial, it compares favorably to a factor of 200 million based on constant discount rates.[34] In other words, the choice of discount rate is rendered less important when one considers the effect of uncertainty.

B. Valuing Future Climate Policy

We can now apply our appropriately adjusted discount factors to our motivating example—the consequences of climate change caused by CO_2 emissions. Starting with the time profile of mitigation benefits shown in Figure 1, we can value those benefits using each of the discount factor series shown in Tables 1 and 2. That is, we take the estimated gain in global output every year in the future due to a one ton reduction in CO_2 emissions in the year 2000, multiply by the discount factor for that year, and add them up. The results of these calculations are shown in Table 3 based on the 4 percent rate that reflects the historic return to government bonds, as well as our high and low sensitivity calculations. For each rate, we report the valuation

Table 3

Expected Discounted Value of Climate Mitigation Benefits (per ton carbon)

		Benefits from 1 ton of carbon mitigation	Relative to constant rate
Government bond rate (4%)	Constant 4% rate	$5.74	—
	Random walk model	$10.44	+82%
	Mean-reverting model	$6.52	+14%
2% rate	Constant 2% rate	$21.73	—
	Random walk model	$33.84	+56%
	Mean-reverting model	$23.32	+7%
7% rate	Constant 7% rate	$1.48	—
	Random walk model	$2.88	+95%
	Mean-reverting model	$1.79	+21%

based on both the random walk and mean-reverting models, as well as for the conventional constant rate model, and indicate the relative effect of uncertainty—the ratio of the valuation under the two uncertainty models to the corresponding valuation with constant rates.[35]

Based on our analysis using government bond rates, uncertainty about future interest rates raises the estimated present value of reducing carbon emissions from about $6 per ton to $10—an increase of more than 80 percent. This assumes that interest rates are best thought of as following a random walk. If we instead assume they always revert back to their long-run average (about 4 percent), we find a more modest effect of about 14 percent.

As one would expect based on the discount factors from Table 2, the relative effect of uncertainty on the present value of expected mitigation benefits is larger when the initial rate and average rate in the future are higher. This reflects the greater opportunity for uncertainty to lower rates when the initial rate is higher (versus a low initial rate where the rate simply cannot go much lower). The effect of uncertainty is a 56 percent increase in discounted mitigation benefits with a 2 percent rate and a 95 percent increase with a 7 percent rate, based on the random walk model. The mean-reverting model again yields a more modest 7 percent increase using the 2 percent rate and a 21 percent increase using the 7 percent initial rate.

Note that while the dollar value of discounted climate benefits is sensitive to the magnitude of the benefits profile we have chosen for illustration, the proportional increase due to incorporation of the effect of discount rate uncertainty depends only on the general shape of the profile. In addition, because we focus on a 400-year horizon, our results are in some sense conservative; extending the horizon further introduces damages that are counted more heavily in the presence of uncertainty.[36] Applying the uncertainty-adjusted discount factors to other GHGs with longer atmospheric residence (e.g., methane (CH_4) or sulfur hexafluoride (SF_6)), or to climate damage profiles that include catastrophic events or other permanent consequences (e.g., species loss), would also generate larger increases in discounted climate damages because the consequences would be more heavily concentrated in the future. In general, the greater the proportion of benefit flows occurring in the distant future, the greater the error introduced through discounting that ignores uncertainty in the discount rate itself.

V. Conclusions

The evaluation of environmental policies frequently requires balancing near-term mitigation costs against long-term environmental benefits. To make these costs and benefits comparable, economic theory suggests discounting future consequences based on the market rate of return to investment. In this way, one gains assurance that environmental policies provide welfare improvements that are at least as good as other productive activities.

Several practical issues complicate the application of this straightforward concept. Current tax policy on corporate income creates a distinction between the rate of return to corporations (greater than 7 percent) and the rate of return that is available to individual investors (below 7 percent). Focusing on the rate available to individuals, there is also a difference between the rate of return to equity (7 percent), the return to bonds (4 percent), and the return to each of these remaining after personal income taxes. While no consensus exists on the appropriate rate to use for discounting, there is a tendency to focus on the return to bonds for evaluating longer-term policies. In the case of policies to mitigate the threat of climate change, the problem is further complicated by a time horizon that involves centuries rather than just years or decades. At these horizons, there are no comparable low-risk market investments that establish future rates of return.

Here, we consider the effect of uncertain future interest rates on the valuation of future benefits, distinct from any uncertainty about the magnitude of future benefits themselves. Because the effect of changes in the discount rate is not symmetric—unexpectedly low discount rates raise valuations by a much larger amount than unexpectedly high discount reduce them—this uncertainty raises future valuations relative to analyses that maintain a fixed discount rate forever.

To best understand how future interest rates are likely to change, we examine the behavior of long-term government bond rates in the United States. These rates reveal persistent changes, including a secular decline from near 6 percent in 1800 to 3 percent at the end of the 20th century along with five noticeable shifts of at least 1 percent lasting ten years or more. This leads us to believe that it should not be surprising if persistent changes in interest rates occur in the future.

We use these data on long-term government bond rates to estimate the uncertainty surrounding interest rates in the past, to simulate uncertain rates in the future, and to compute the appropriate discount factors for various time horizons and alternative base rates. Starting with the assumption that the current rate should equal the average rate of return to government bonds (4 percent), but that future rates follow an uncertain random walk estimated from the data, we find that the discount factor after 400 years is over 40,000 times higher than if we instead assume that the rate remains fixed at 4 percent. When we construct similar measures of the uncertainty effect for alternative rates of 2 percent and 7 percent, we find that the effect of uncertainty is larger for higher interest rates—this makes sense because higher rates have a larger opportunity to decline with uncertainty and thereby raise valuations. Comparing the discount factors directly, we find that the valuation of benefits occurring in the future is less sensitive to the choice of the current discount rate when the effect of uncertainty is taken into account. That is, not only do valuations rise when one considers uncertainty, but they become less sensitive to whether the analysis is based on the after-tax return on bonds or the pre-tax return on equities.

Applied to one estimate of the consequences of climate change, the effect of uncertainty is large. Using the government bond rate of 4 percent, the expected present value of damages from current CO_2 emissions increases by over 80 percent when we incorporate the effect of future interest rate uncertainty. An initial rate of 7 percent yields a 95 percent increase, while an initial rate of 2 percent yields an increase of about 55 percent.

There is a frequent tendency among both economists and non-economists to be skeptical of using conventional discounting techniques to value benefits over hundreds of years because they render future benefits insignificant and somehow that seems “wrong.” The results of this study suggest that this concern is at least partially addressed—without abandoning conventional economic theory—by viewing future interest rates as uncertain. While this will not yield the same dramatic effects as the decision to arbitrarily apply a lower discount rate, uncertainty does have a large effect on valuations at horizons of 100 years or more in the future.

Endnotes

1. For example, the lifespan of an electricity-generating unit is typically 50 to 70 years.

2. The duration may be even longer if climate change leads to a fundamental and irreversible shift in the Earth's climate systems.

3. See Weitzman (1998).

4. We use this particular path of benefits not because we believe it is the "best" estimate of climate damages, but rather because the model is relatively simple and transparent, has been used in a wide range of analyses, and exhibits a time profile of damages that is reasonable. As we describe further below, the quantitative effect of uncertainty on the percentage increase in discounted climate damages is dependent only on the shape of the path of damages, not its magnitude. This robustness is important because there is little consensus on the consequences of climate change, and many have argued that the DICE model is somewhat conservative in its damage estimates (Repetto and Austin, 1997). For example, one possibility ignored by the DICE model (and most other models) is that GHG emissions may lead to a permanent shift in climatic patterns that will remain even if atmospheric concentrations of these gases eventually decline (Broecker, 1997).

5. Note that while the damages associated with a single ton of emissions declines over time, the cumulative effect of all GHG emissions is not projected to decline over the next several hundred years.

6. The real return to large company stocks was 8 percent over the period 1926-98 (7 percent over 1926-90). The real return to corporate bonds was 2.7 percent and to government bonds was 2.2 percent over the same period (Ibbotson Associates, 1999). Based on *ex ante* survey measures of inflation such as those discussed in Thomas (1999), rather than the *ex post* realized rate used by Ibbotson, we find real expected rates of return on government bonds closer to 4 percent (see Box 5 on historical interest rates and Appendix A on estimating historic returns).

7. Another issue arises when the costs and benefits of a project are measured in *nominal* rather than *real* terms. It is then necessary to use nominal discount rates that are typically several percent higher due to inflation. That is, when we talk about $100 of benefits in 2050, do we imagine that this $100 can buy a similar basket of goods as it can buy today (a constant-dollar benefit measure), or do we imagine that this $100 is worth a lot less in 2050 due to inflation (a nominal benefit measure)? Because it is straightforward to establish whether a cost and benefit analysis is based on real or nominal measures, this distinction is relatively uncontroversial. Generally, most analyses utilize real measures.

8. See U.S. EPA (2000, p. 47).

9. Thus, a standard approach to policy analysis under uncertainty is to convert uncertain cost and benefit flows to certainty equivalents and then discount these flows using a risk-free (or at least low-risk) rate of return (see, e.g., Arrow et al., 1996; Lind, 1982).

10. See Nordhaus (1994, p.129).

11. See Lind (1982) for a discussion of the shadow price of capital approach. The choice between investment and consumption rates of return is actually a bit more involved than the preceding paragraphs suggest. When public projects directly offset or "crowd out" private investment (e.g., by raising money through bond offerings), it is easy to see why they ought to offer at least as high a return as the foregone private investment. If public projects achieve such a return, and the return accrues to firms, firms are clearly better off. When public projects offset consumption (or when costs and benefits are both measured in terms of their consumption consequences), one is no longer measuring improvements in terms of firm profit, but in terms of consumer welfare, and the consumption rate of interest is appropriate.

12. While the consumption rate of interest measures the rate at which changes in consumption *can* be moved across time, one cannot be sure that it is the return *required* for a welfare improvement unless we assume the existing consumption levels were optimally chosen in response to the consumption interest rate. Although one might take it for granted that businesses regularly maximize profits—and that blindly crowding out private investment will lower profits—there is some evidence that consumers behave irrationally (e.g., forced savings might raise welfare; see Box 4 for further discussion). Nonetheless, when public investment or regulation influences consumption, there are no good alternatives to assuming that the consumer rate of interest reflects consumer preferences for tradeoffs over time.

13. In the case of equities, one really only observes *past* rates.

14. An exception is the British *consuls* (first issued in 1749), which are in principle issued in perpetuity—holders can never claim their principal (the debt can be retired at the option of the government, but this is unlikely given the extremely low nominal rate). However, there are several problems with using yields on the consuls as a reliable indicator of future interest rates—most importantly, they provide a single summary statistic about all future rates rather than information about specific rates in specific years. Homer and Sylla (1998, pp. 160, 443) discuss other problems with using the yield on consuls as a reliable measure of the market rate.

15. Even if the rate of discount on utility is zero, the rate of discount on consumption may be greater than zero because future generations are likely to be richer and typically consumption becomes less valuable at the margin as this occurs. In other words, an extra $1 of consumption by the current generation could be more valuable than an extra $1 of consumption by future generations because the current generation is relatively poor and $1 is more valuable to poorer people. This is the distinction between utility discounting and consumption discounting raised by some authors including Nordhaus (1994).

16. Arrow et al. (1996) and Cline (1999), among others, discuss this so-called "prescriptive" approach.

17. Bazelon and Smetters (1999) describe this approach in their survey of "Discounting Inside the Beltway." We discuss the problems inherent with this approach in Box 4, Time Inconsistency and Hyperbolic Discounting. Arrow (1999) discusses one approach to thinking about this dilemma, viewing it as a game played between generations.

18. Of course, there could be considerable disagreement about that ethical rate, leading to a different type of uncertainty discussed by Weitzman (1999).

19. Arguably, in that case the uncertainty should be quantified using historic data on consumption growth, not interest rates.

20. See Lind (1982), a special 1990 issue of the *Journal of Environmental Economics and Management* (Volume 18, Number 2) devoted to discounting, and most recently, Portney and Weyant (1999).

21. To convert nominal to real rates, we use a ten-year moving average of the one-year inflation forecast collected as part of the Livingston Survey since 1945 (Thomas, 1999). Note that prior to 1945, there is little evidence of persistent inflation because the value of money was tied to a gold standard. There were some years of high inflation, usually associated with wars or other temporary disruptions, but they were always offset by subsequent years of deflation. In fact, price levels as measured by the Consumer Price Index were the same in the early 1940s as they were in 1800 (U.S. Bureau of the Census 1975, p. 211). As a result, financial market participants anticipated that the future rate of inflation would be about zero.

22. To some extent, recent fluctuations appear different—and perhaps more volatile. This may be the result of our difficulty converting from nominal to real measures in the face of rapidly changing inflation expectations. Errors from this conversion could introduce extra variation but would tend to vanish when averaged over time. On the other hand, a more active policy by the Federal Reserve could be causing sharper changes in real rates.

23. This point, made recently by Weitzman (1998), inspired our work.

24. Note that while the result that the uncertainty-adjusted discount rate should decline in the future appears similar to the approach of Cline (1999) and others of using lower future discount rates, the basis for the result and the empirical method of applying it are quite different.

25. See "A Positive Rate of Interest" in Mandler (1999).

26. In Nordhaus (1994), for example, the interest rate depends on the equilibrium marginal product of capital.

27. The posterior odds ratio, a measure of the relative likelihood of one model over another based on observed data, is about 1.5 in favor of the random walk model (i.e., 60% ÷ 40%=1.5). See Schwarz (1978).

28. The values +/–0.06 approximate the estimated variance of the data assuming a simple random walk in logs.

29. The actual real bond return today is about 3 percent.

30. The idea that the value associated with an uncertain outcome—in this case the path of interest rates—equals the average or expected value is described by expected utility theory. See Chapter 6 of Mas-Colell et al. (1995).

31. If the valuation in the future itself is uncertain, the calculation may not be valid if the uncertainty about the discount factor is correlated with the uncertainty about the future valuation.

32. A rate of 4 percent reflects the approximate 200-year average as well as the average over the past 20 years. It also falls close to the middle of the range of defensible consumption rates of interest (2-7 percent).

33. Note that because the logged interest rate follows a random walk, it implies that the disturbances to the (unlogged) interest rate are scaled by the magnitude of the interest rate—larger rates lead to larger disturbances, and vice-versa for smaller initial rates. Casual observation of historic fluctuations supports such an assumption (see figures on pages 369, 394, and 424 of Homer and Sylla, 1998).

34. Discounting at a constant rate of 7 percent, $100 delivered 400 years in the future would be valued at $\$2 \times 10^{-10}$ today, a fact obscured due to rounding in the bottom line of Table 2, column 6. Note that $0.04 \div 2 \times 10^{-10} = 2 \times 10^{8}$ or 200 million.

35. For brevity, we did not report the mean-reverting discount factors based on initial rates of 2 percent and 7 percent in Table 2.

36. We have estimated that this understatement might be as much as 4 percent based on the rate of decline in discounted carbon benefits at the end of our 400-year horizon.

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Appendix A: Estimation

As described in the text, we model our prediction of future interest rates based solely on observations of past interest rates. We also build our model around logged interest rates to eliminate the possibility of eventually predicting negative values.[1] Even within the confines of this simple model, at least three issues arise: (1) the number of past values to use in the prediction, (2) whether to include a trend, and (3) whether to assume interest rates are mean-reverting or a random walk.

The first question is a long-standing question in statistics connected with determining the "order" of a model—that is, how many parameters (past values in our interest rate problem) are justified based on the added information provided by those parameters. A common approach is to pick the version of the model that maximizes the Schwarz-Bayes Information Criteria (SBIC). These criteria, discussed by Schwarz (1978), can be interpreted as the probability that a particular version of the model is correct. Using this criterion, we find that logged interest rates are best predicted by the three most recent past values.

The second question is suggested by the fact that rates have declined over the past 200 years (Figure 4) and indeed over the past millennium (Box 5). If we estimate a simple model that includes a trend, we do find that it is statistically significant. Unfortunately, a well-known feature of random walks and other highly-persistent data series is that one may "find" a trend despite the lack of trend in the true underlying behavior (as pointed out by Nelson and Plosser, 1982). If one thinks about the path taken by a drunk—frequently taken as an example of a random walk—it will most likely show some trend over time. In fact, the odds that the drunk will end up exactly where he started would seem to be virtually zero. When we account for this fact, we find that the trend is not statistically significant.[2]

Another reason we remain skeptical of a trend in the data is its implication for the future. At some level, we know that a negative trend in interest rates would have to eventually stop because interest rates cannot

[1] Note that unlogging (or exponentiating) numbers always produces positive values.

[2] Nonetheless, we still need to include a "trend correction" in the random walk model so that the expected rate remains constant. This correcting trend is constrained to be equal to $-\sigma_{\xi}^{2}/(2(1+\rho_2+2\rho_3)^2)$, which exactly offsets the positive trend that would otherwise exist in the rate due to an increasing variance in the logged rate over time. Inclusion of this trend correction turns out to have no significant effect on the parameter estimates.

be negative. The question is then: When does it stop? Posed in this way, it seems almost negligent to simply fit a trend in the logged data and pretend that the future path can be confidently extrapolated. Instead, we find it more appealing to imagine that the past trend was the result of previous random movements.

The final question is perhaps the most difficult because there is still so much controversy in the economics literature over whether macroeconomic variables, such as the interest rate, should be thought of as random walks or whether they revert to long-run trends. This distinction turns out to have significant consequences for our analysis because it alters the possibility of consistently low interest rates in the distant future. If we treat the mean interest rate over the past 200 years as the long-run mean in all future scenarios, this substantially reduces the probability of low interest rate paths of the sort highlighted by the bottom path in Figure 5. While we believe that the random walk model is more appealing because it allows the future to follow the patterns—but not necessarily the precise experience of the past—we present both results.

We therefore specify the following stochastic model of interest rate behavior. The interest rate in period t, r_t, is uncertain, and its uncertainty has both a permanent component η, and a more fleeting yet persistent component ε_t:

$$r_t = \eta \exp(\varepsilon_t), \tag{1}$$

or after taking logs,

$$\ln r_t = \ln \eta + \varepsilon_t, \tag{2}$$

where η is a log-normally distributed random variable with mean $\bar{\eta}$ and variance σ^2_η. Shocks to the interest rate, ε_t, follow the autoregressive form

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \rho_3 \varepsilon_{t-3} + \xi_t, \tag{3}$$

where the correlation parameters ρ describe the persistence of deviations from the mean rate, and ξ_t is an independent identically distributed, mean-zero, normally distributed random variable with variance σ^2_ξ.[3] The two uncertain components η and ε_t are assumed to be independent. Thus, the mean of r_t is $\bar{\eta}$. This

[3] A normality test on the data rejects a normal distribution for the data in levels but does not reject normality once the data have been logged.

is called the "mean-reverting" model because, with $\rho<1$, the series would eventually tend toward its long-run mean.

We also estimate the random walk version of Equation (2), which is given by

$$\ln r_t = \ln r_o + \varepsilon_t, \tag{4}$$

where now the constraint is imposed that $\rho_1+\rho_2+\rho_3=1$ for the autoregressive shocks ε_t in Equation (3). Under this assumption, it can be shown that a shock ξ_t in period t will have a permanent effect $\xi_t/(1+\rho_2+2\rho_3)$ on future values of ε_{t+s} (after a suitable time s has elapsed). Note that r_o in the random walk model replaces η in the mean-reverting model because interest rates are now modeled as an accumulation of permanent innovations from some initial rate (r_o), rather than deviations from a long-run mean (η).

We estimate the model parameters conditional on the initial observations, dropping those for which lagged values are not directly observed. With a single lagged value in the autoregression, this is equivalent to the Cochrane-Orcutt (1949) method; with more lagged values, this approach is referred to as conditional maximum likelihood (Hamilton, 1994). Table A-1 presents the results of both the random walk and mean-reverting models, with and without time trends. As noted earlier, standard tests fail to reject the random walk restriction.[4] Note also that the trend, which appears significant in the mean-reverting model, is no longer significant when the random walk is assumed. The parameter estimates are similar in all four models.

[4]The standard test, an augmented Dickey-Fuller (1979) test, rejects both the trend and no-trend models at the 5 percent level.

Table A-1

Results for **Econometric Models** of Interest Rates

$$\ln r_t = \ln \eta + \varepsilon_t \text{ and } \varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \rho_3 \varepsilon_{t-3} + \xi_t$$

	random walk model ($\sum \rho_s = 1$)		Mean-reverting model	
mean rate ($\bar{\eta}$)			3.69*[c]	3.95*[c]
std error (σ_η)			0.45	0.23
autoregressive coefficients				
ρ_1	1.92* (0.06)	1.92* (0.06)	1.88* (0.07)	1.85* (0.07)
ρ_2	–1.34* (0.12)	–1.34* (0.12)	–1.31* (0.12)	–1.26* (0.12)
ρ_3	0.43* (0.07)	0.43* (0.07)	0.40* (0.07)	0.36* (0.07)
trend	[b]	–0.0037[a] (0.0055)		–0.0033*[a] (0.0010)
σ_ξ^2	0.0015* (0.0002)	0.0015* (0.0002)	0.0015* (0.0002)	0.0015* (0.0002)

*Significant at the 5 percent level.

[a] Indicates a linear time trend estimated alongside ε_t in each model.

[b] As noted in footnote 2, the parameter estimates are unaffected by either constraining the trend to be zero or constraining it to equal $-\sigma_\xi^2/(2(1+\rho_2+2\rho_3)^2) = -0.003$, which exactly offsets the positive trend that would otherwise exist in the actual (unlogged) rate due to an increasing variance in the logged rate over time.

[c] The mean rates in this table were constructed, for simulation purposes, to reflect a continuously compounded rate. To convert to simple annual rates, simply compute $100 \times (\exp(\eta/100) - 1)$, e.g., $100 \times (\exp(3.69/100) - 1) = 3.76$.

Appendix B: Simulations

As described in the text, we simulate the model by drawing shocks to the logged interest rate each period and constructing paths as shown in Figure 6. The actual simulations are only slightly more complex than portrayed by the diagram.

First, the simulated shocks ξ_t assume a normal distribution with variance σ^2_ξ (given in Table A-1) rather than the dichotomous values +/–0.06 with equal probability as provided in the simplified example. Second, we allow for uncertainty in all the parameters of the model: ρ_1, ρ_2, ρ_3, η, and σ^2_ξ. The first four are assumed to be jointly normal and the last is independently normal (the covariance, not reported in Table A-1, is computed from the data). These parameters are drawn first, followed by the sequence of uncorrelated ξ_t, which are then converted via Equation (3) into correlated shocks ε_t. This creates a single interest rate path, analogous to Figure 6. This process is then repeated 100,000 times to construct the expected discount factors reported in Tables 1 and 2, and used to compute the climate benefit estimates in Table 3.

A final adjustment is necessary when we want to compare the effect of uncertainty to a model based on a constant discount rate. While the expectation of ε_t equals zero in the formula for the logged interest rate, it does not follow that $E\left[\exp(\varepsilon_t)\right]=\exp\left(E[\varepsilon_t]\right)=1$ when we exponentiate the expression in Equation (2) to obtain the actual (unlogged) interest rate in Equation (1). Instead, a standard result for a normal mean zero error ε_t is that $E\left[\exp(\varepsilon_t)\right]=\exp\left(\sigma^2_{\varepsilon_t}/2\right)>1$ where $\sigma^2_{\varepsilon_t}$ is the variance of ε_t and will generally depend on the time period because the variance grows over time. Without any adjustment we are not only introducing uncertainty, we are simultaneously raising the expected interest rate by a factor of $\exp\left(\sigma^2_{\varepsilon_t}/2\right)$—a problem if our interest is solely identifying the effect of uncertainty about a constant mean.

We employ different approaches in the mean-reverting and random walk models to fix this problem. In the random walk model, this factor $\exp(\sigma^2_{\varepsilon_t}/2)$ has a limiting value of $\exp(t \times \sigma^2_\xi/(2(1+\rho_2+\rho_3)^2))$ that can be eliminated with an opposing trend of $-\sigma^2_\xi/(2(1+\rho_2+\rho_3)^2) \times t$ in Equation (4) (see footnote 2 in Appendix A). By estimating the model constrained with this trend, our simulations will have a constant expected value for the interest rate. This constraint is not rejected by the data; in fact, the parameter estimates and standard errors reported in the first column of Table A-1 do not change the degree of accuracy reported in the table (the implied trend is $-\sigma^2_\xi/(2(1+\rho_2+2\rho_3)^2) = -0.003$).

The mean-reverting model is more complex. Depending on how close the randomly drawn parameters ρ_1, ρ_2, and ρ_3, are to a random walk (where $\Sigma_s \rho_s = 1$), it may take many years before the variance of ε_t approximates its long-run value (a function of the ρ_s's and σ^2_ξ, but not t). Therefore, we cannot subtract off a single adjustment in all periods, as we did with the random walk. For that reason, we separately compute the expected variance $\sigma^2_{\varepsilon_t}$ each period based on the particular values of the ρ_s's and σ^2_ξ, and subtract that value (divided by two) from the simulated value of ε_t, so we have $E[\exp(\varepsilon_t - \sigma^2_{\varepsilon_t}/2)] = \exp(\sigma^2_{\varepsilon_t}/2 - \sigma^2_{\varepsilon_t}/2) = 1$.

notes

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notes

This report explores some of the analytic difficulties of applying conventional discounting techniques to long-term problems such as global climate change. The Pew Center on Global Climate Change was established by the Pew Charitable Trusts to bring a new cooperative approach and critical scientific, economic, and technological expertise to the global climate change debate. We intend to inform this debate through wide-ranging analyses that will add new facts and perspectives in four areas: policy (domestic and international), economics, environment, and solutions.

Pew Center on Global Climate Change
2101 Wilson Boulevard
Suite 550
Arlington, VA 22201
Phone (703) 516-4146
www.pewclimate.org